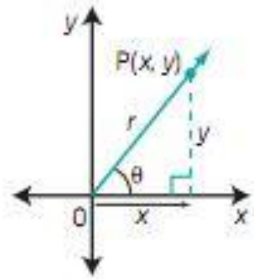
Math 20-1 Topic 2.2 Notes & Class Examples Trig Ratios of Any Angle

Finding the Trigonometric Ratios of Any Angle θ, where 0° ≤ θ < 360°

Suppose θ is any angle in standard position, and P(x, y) is any point on its terminal arm, at a distance r from the origin. Then, by the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$.



You can use a reference triangle to determine the three primary trigonometric ratios in terms of x, y, and r.

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\quad\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

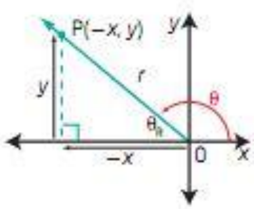
$\sin\theta = \dfrac{y}{r}$ $\quad\cos\theta = \dfrac{x}{r}$ $\quad\tan\theta = \dfrac{y}{x}$

The chart below summarizes the signs of the trigonometric ratios in each quadrant. In each, the horizontal and vertical lengths are considered as directed distances.

Quadrant II
$90° < \theta < 180°$

$\sin\theta = \dfrac{y}{r}$ $\quad\cos\theta = \dfrac{-x}{r}$ $\quad\tan\theta = \dfrac{y}{-x}$

$\sin\theta > 0$ $\quad\cos\theta < 0$ $\quad\tan\theta < 0$

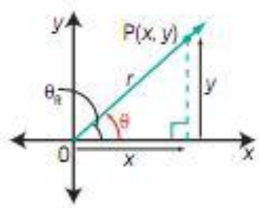


$\theta = 180° - \theta_R$

Quadrant I
$0° < \theta < 90°$

$\sin\theta = \dfrac{y}{r}$ $\quad\cos\theta = \dfrac{x}{r}$ $\quad\tan\theta = \dfrac{y}{x}$

$\sin\theta > 0$ $\quad\cos\theta > 0$ $\quad\tan\theta > 0$



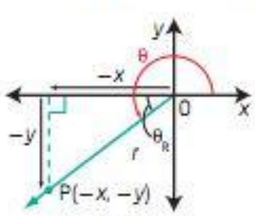
$\theta = \theta_R$

Why is r always positive?

Quadrant III
$180° < \theta < 270°$

$\sin\theta = \dfrac{-y}{r}$ $\quad\cos\theta = \dfrac{-x}{r}$ $\quad\tan\theta = \dfrac{-y}{-x}$

$\sin\theta < 0$ $\quad\cos\theta < 0$ $\quad\tan\theta > 0$

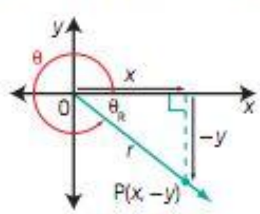


$\theta = 180° + \theta_R$

Quadrant IV
$270° < \theta < 360°$

$\sin\theta = \dfrac{-y}{r}$ $\quad\cos\theta = \dfrac{x}{r}$ $\quad\tan\theta = \dfrac{-y}{x}$

$\sin\theta < 0$ $\quad\cos\theta > 0$ $\quad\tan\theta < 0$



$\theta = 360° - \theta_R$

Example 1

Write Trigonometric Ratios for Angles in Any Quadrant

The point $P(-8, 15)$ lies on the terminal arm of an angle, θ, in standard position. Determine the exact trigonometric ratios for $\sin \theta$, $\cos \theta$, and $\tan \theta$.

Your Turn

The point $P(-5, -12)$ lies on the terminal arm of an angle, θ, in standard position. Determine the exact trigonometric ratios for $\sin \theta$, $\cos \theta$, and $\tan \theta$.

Example 2

Determine the Exact Value of a Trigonometric Ratio

Determine the exact value of $\cos 135°$.

Your Turn

Determine the exact value of $\sin 240°$.

Example 3

Determine Trigonometric Ratios

Suppose θ is an angle in standard position with terminal arm in quadrant III, and $\cos \theta = -\frac{3}{4}$. What are the exact values of $\sin \theta$ and $\tan \theta$?

Your Turn

Suppose θ is an angle in standard position with terminal arm in quadrant III, and $\tan \theta = \frac{1}{5}$. Determine the exact values of $\sin \theta$ and $\cos \theta$.

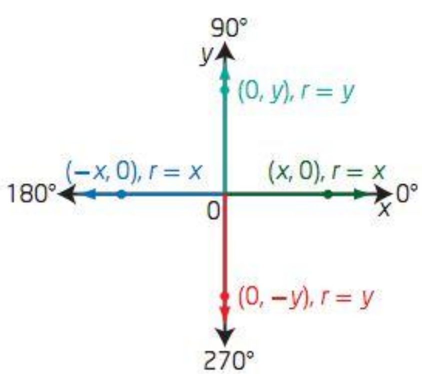
Example 4

Determine Trigonometric Ratios of Quadrantal Angles

Determine the values of sin θ, cos θ, and tan θ when the terminal arm of **quadrantal angle** θ coincides with the positive y-axis, $\theta = 90°$.

Your Turn

Use the diagram to determine the values of sin θ, cos θ, and tan θ for quadrantal angles of 0°, 180°, and 270°. Organize your answers in a table as shown below.



	0°	90°	180°	270°
sin θ		1		
cos θ		0		
tan θ		undefined		

Solving for Angles Given Their Sine, Cosine, or Tangent

Step 1 Determine which quadrants the solution(s) will be in by looking at the sign (+ or −) of the given ratio.

Step 2 Solve for the reference angle.

Why are the trigonometric ratios for the reference angle always positive?

Step 3 Sketch the reference angle in the appropriate quadrant. Use the diagram to determine the measure of the related angle in standard position.

Example 5

Solve for an Angle Given Its Exact Sine, Cosine, or Tangent Value

Solve for θ.

a) $\sin\theta = 0.5,\ 0° \le \theta < 360°$

b) $\cos\theta = -\frac{\sqrt{3}}{2},\ 0° \le \theta < 180°$

Your Turn

Solve $\sin\theta = -\frac{1}{\sqrt{2}},\ 0° \le \theta < 360°$.

Example 6

Solve for an Angle Given Its Approximate Sine, Cosine, or Tangent Value

Given $\cos \theta = -0.6753$, where $0° \leq \theta < 360°$, determine the measure of θ, to the nearest tenth of a degree.

Your Turn

Determine the measure of θ, to the nearest degree, given $\sin \theta = -0.8090$, where $0° \leq \theta < 360°$.

Key Ideas

- The primary trigonometric ratios for an angle, θ, in standard position that has a point P(x, y) on its terminal arm are $\sin \theta = \frac{y}{r}$, $\cos \theta = \frac{x}{r}$, and $\tan \theta = \frac{y}{x}$, where $r = \sqrt{x^2 + y^2}$.
- The table show the signs of the primary trigonometric ratios for an angle, θ, in standard position with the terminal arm in the given quadrant.

	Quadrant			
Ratio	**I**	**II**	**III**	**IV**
sin θ	+	+	−	−
cos θ	+	−	−	+
tan θ	+	−	+	−

- If the terminal arm of an angle, θ, in standard position lies on one of the axes, θ is called a quadrantal angle. The quadrantal angles are 0°, 90°, 180°, 270°, and 360°, $0° \leq \theta \leq 360°$.