

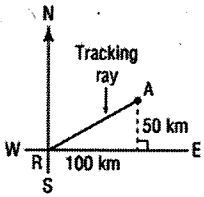
Math 20-1 Lesson 2.1 Angles in Standard Position in Quadrant 1

FOCUS Relate the primary trigonometric ratios to angles in standard position.

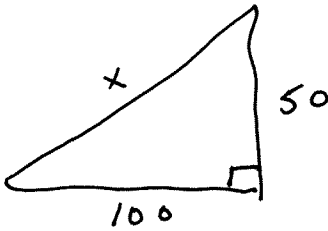
Get Started

Air traffic controllers use radar to track aircraft. A tracking ray rotates about the centre of the radar display and shows a "blip" as it identifies an aircraft's position. The blip indicates the distance and direction of the aircraft from the radar antenna.

One aircraft, A, is 100 km due east and 50 km due north of a radar antenna, R.



To the nearest kilometre, what is the distance between the aircraft and the radar antenna?



$$a^2 + b^2 = c^2$$

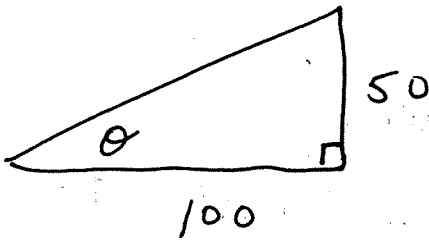
$$x^2 = 50^2 + 100^2$$

$$x^2 = 12500$$

$$x = \sqrt{12500}$$

$x \doteq 111.8$ or **112 km**

What is the angle between the tracking ray to the aircraft and due east?



Recall soh cah toa

$$\tan \theta = \frac{50}{100}$$

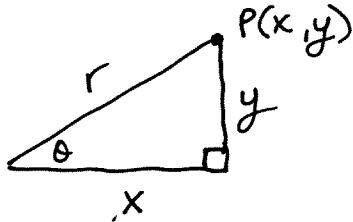
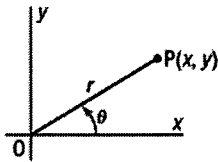
$$\tan^{-1} \left(\frac{50}{100} \right) \doteq 27^\circ$$

Construct Understanding

Use this diagram.

Write an expression for:

- r in terms of x and y
- the value of θ in terms of x and y
- the x -coordinate of P in terms of r and θ
- the y -coordinate of P in terms of r and θ



$$a) \begin{aligned} r^2 &= x^2 + y^2 \\ r &= \sqrt{x^2 + y^2} \end{aligned}$$

$$b) \tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$c) \cos \theta = \frac{x}{r}$$

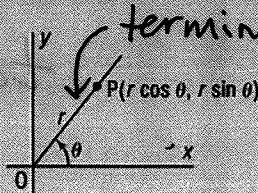
$$x = r \cos \theta$$

$$d) \sin \theta = \frac{y}{r}$$

$$y = r \sin \theta$$

The coordinates of a point P on the coordinate plane can be described by its distance r from the origin, O , and the angle θ that OP makes with the positive x -axis. When the angle θ , between 0° and 360° , is measured counterclockwise from the positive x -axis, the angle is in **standard position**. The ray OP is the **terminal arm** of the angle and the point P is a **terminal point** for the angle.

Angle in Standard Position in Quadrant 1

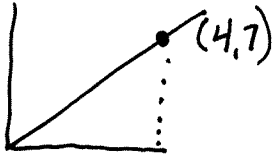


in Standard Position
because the angle
is measured from
the positive x -axis.

Example 1

The point P(4, 7) is on the terminal arm of an angle θ in standard position.

- Determine the distance r from the origin to P.
- Determine the primary trigonometric ratios of θ .
- Determine the measure of θ to the nearest degree.



a)

$$a^2 + b^2 = c^2$$

$$4^2 + 7^2 = c^2$$

$$16 + 49 = c^2$$

$$\sqrt{65} = c$$

b)

$$\sin \theta = \frac{7}{\sqrt{65}}$$

$$\cos \theta = \frac{4}{\sqrt{65}}$$

$$\tan \theta = \frac{7}{4}$$

c)

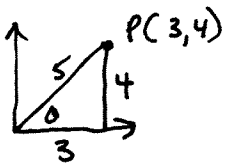
$$\sin^{-1}\left(\frac{7}{\sqrt{65}}\right) \approx 60.25^\circ$$

$$\cos^{-1}\left(\frac{4}{\sqrt{65}}\right) \approx 60.25^\circ$$

$$\tan^{-1}\left(\frac{7}{4}\right) \approx 60.25^\circ$$

1. The point P(3, 4) is on the terminal arm of an angle in standard position.

- Determine the distance r from the origin to P.
- Determine the primary trigonometric ratios of θ .
- Determine the measure of θ to the nearest degree.



$$\sin \theta = \frac{4}{5}$$

$$\sin^{-1}\left(\frac{4}{5}\right) = 53^\circ$$

$$\cos \theta = \frac{3}{5}$$

$$\cos^{-1}\left(\frac{3}{5}\right) = 53^\circ$$

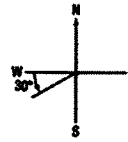
$$\tan \theta = \frac{4}{3}$$

$$\tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$$

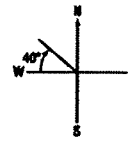
1) $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$ $\sin \theta = \frac{4}{5}$, $\cos \theta = \frac{3}{5}$, $\tan \theta = \frac{4}{3}$ c) 53°

Trigonometry is essential to navigation. A direction can be described by relating it to two of the compass points: north, south, west, and east

For example, a heading of $W30^\circ S$ means from a direction due west, rotate 30° counterclockwise; that is, toward south.

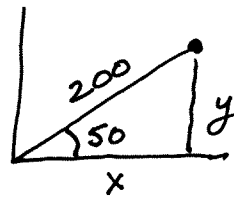


Similarly, a heading of $W40^\circ N$ means from a direction due west, rotate 40° clockwise; that is, toward north.



Example 2

An aircraft made an emergency landing 200 km from an airport. Its heading from the airport was $E50^\circ N$. The land-based rescue team has to travel east then north to get to the aircraft. To the nearest kilometre, how far should the team travel in each direction?



$$x = r \cos \theta$$

$$= 200 \cos 50$$

$$= 128.55\dots$$

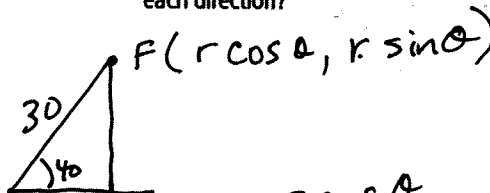
$$y = r \sin \theta$$

$$= 200 \sin 50$$

$$= 153.20\dots$$

The team should travel 129 km east and 153 km North.

2. A forest ranger sees smoke rising from a point that lies in a direction $E40^\circ N$. She estimates that the distance from the ranger station is about 30 km. The firefighters at the ranger station have to travel east then north to get to the fire. To the nearest kilometre, how far should the firefighters travel in each direction?



$$x = r \cos \theta$$

$$x = 30 \cos 40$$

$$x = 23 \text{ km E}$$

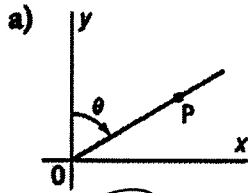
$$y = r \sin \theta$$

$$y = 30 \sin 40$$

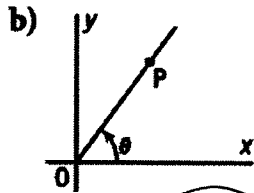
$$y = 19 \text{ km N}$$

23 E 19 N

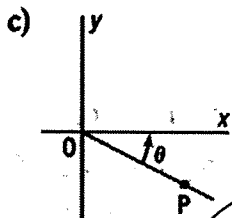
3. State whether each diagram represents an angle in standard position. Explain your thinking.



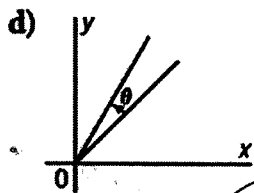
no



yes



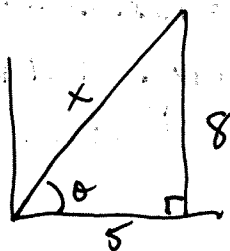
no



no

4. Point P(5, 8) is on the terminal arm of an angle θ in standard position.

a) Sketch the angle.



b) Determine the distance from the origin to P.

$$\begin{aligned} x^2 &= 5^2 + 8^2 \\ &= 25 + 64 \\ x &= \sqrt{89} \end{aligned}$$

c) Write the primary trigonometric ratios of θ .

$$\sin \theta = \frac{8}{\sqrt{89}} \quad \theta = 58^\circ$$

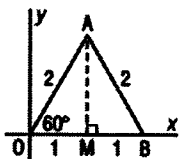
$$\cos \theta = \frac{5}{\sqrt{89}} \quad \theta = 58^\circ$$

$$\tan \theta = \frac{8}{5} \quad \theta = 58^\circ$$

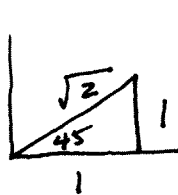
d) What is the measure of θ to the nearest degree?

58°

5. a) Use this diagram to determine the exact primary trigonometric ratios of 60° .



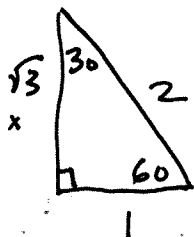
You will need the results of question 5 in Lesson 6.2.



$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = 1$$



$$\begin{aligned} 1^2 + x^2 &= 2^2 \\ x^2 &= 2^2 - 1^2 \\ x^2 &= 4 - 1 \\ x^2 &= 3 \\ x &= \sqrt{3} \end{aligned}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} \text{ or } \sqrt{3}$$

- b) Use the diagram in part a to determine the exact primary trigonometric ratios of 30° .

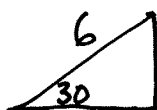
$$\sin 30^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

- c) How are the values of the primary trigonometric ratios of 30° and 60° related? How can you predict the relationship by inspecting the triangles?

→ the sin + cos are reversed
and tan 30 and 60 are reciprocals.

6. For each angle below, determine the exact coordinates of a point on the terminal arm of the angle in standard position.

a) 30°



if $r = 6$

(x, y)

$$(r \cos 30, r \sin 30)$$

$$\left(6 \left(\frac{\sqrt{3}}{2} \right), 6 \left(\frac{1}{2} \right) \right)$$

$$(3\sqrt{3}, 3)$$

b) 45°

$$\sin 45 = \frac{1}{\sqrt{2}} \quad \cos 45 = \frac{1}{\sqrt{2}} \quad \tan 45 = 1$$

$$(r \cos 45, r \sin 45)$$

$$\left(6 \left(\frac{1}{\sqrt{2}} \right), 6 \left(\frac{1}{\sqrt{2}} \right) \right) = \left(\frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}} \right)$$

c) 60°

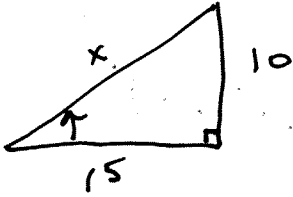
$$(r \cos 60, r \sin 60)$$

$$6 \left(\frac{1}{2} \right), 6 \frac{\sqrt{3}}{2}$$

$$(3, 3\sqrt{3})$$

7. A support cable is anchored 15 m from the base of a pole and is attached to the pole 10 m above the ground.

a) Determine the length of the cable to the nearest tenth of a metre.



$$\begin{aligned} x^2 &= 10^2 + 15^2 \\ x^2 &= 100 + 225 \\ x^2 &= 325 \\ x &= \sqrt{325} \\ x &\approx 18.0277... \\ \mathbf{x &\approx 18 \text{ m}} \end{aligned}$$

b) To the nearest degree, what angle does the cable make with the ground?

$$\tan \theta = \frac{10}{15}$$

$$\tan^{-1} \left(\frac{10}{15} \right)$$

$$\theta = 33.69 \approx \mathbf{34^\circ}$$

8. a) Determine the distance of each point from the origin.

i) A(4, 6)

ii) B(7, 3)

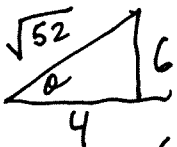
$$\begin{aligned} r^2 &= 4^2 + 6^2 \\ r^2 &= 16 + 36 \\ \mathbf{r &= \sqrt{52}} \end{aligned}$$

$$\begin{aligned} r^2 &= 7^2 + 3^2 \\ r^2 &= 49 + 9 \\ \mathbf{r &= \sqrt{58}} \end{aligned}$$

b) Each point in part a is on the terminal arm of an angle θ in standard position. For each angle, determine $\cos \theta$, $\sin \theta$, $\tan \theta$, and the measure of θ to the nearest degree.

i) A(4, 6)

ii) B(7, 3)

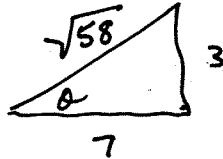


$$\tan \theta = \frac{6}{4}$$

$$\sin \theta = \frac{6}{\sqrt{52}}$$

$$\theta = 56^\circ$$

$$\cos \theta = \frac{4}{\sqrt{52}}$$



$$\sin \theta = \frac{3}{\sqrt{58}}$$

$$\tan \theta = \frac{3}{7}$$

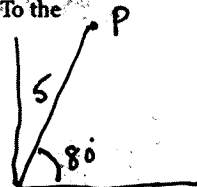
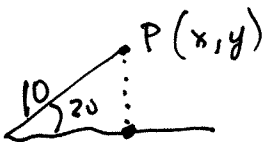
$$\cos \theta = \frac{7}{\sqrt{58}}$$

$$\theta = 23^\circ$$

9. Point P(x, y) is on the terminal arm of each angle below in standard position. The distance r between P and the origin is given. To the nearest tenth, determine the coordinates of P.

a) 20° ; $r = 10$

b) 80° ; $r = 5$



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x = 10 \cos 20 = \mathbf{9.4}$$

$$y = 10 \sin 20 = \mathbf{3.4}$$

$$P(9.4, 3.4)$$

$$x = r \cos \theta$$

$$x = 5 \cos 80 = \mathbf{0.87}$$

$$y = r \sin \theta$$

$$y = 5 \sin 80 = \mathbf{4.9}$$

$$P(0.87, 4.9)$$