8 Chapter Review

FREQUENTLY ASKED Questions

Q: What are scale diagrams and scale models, and what are they used for?

A: A scale diagram is a drawing that is a reduction or an enlargement of a 2-D shape or 3-D object. A scale model is a reduced or enlarged model of a 3-D object. A scale diagram or scale model is similar to the actual shape or object. To create a scale diagram or model, the same scale factor is applied to all the linear measurements.

For example, to create a model of a train using a scale ratio of 1 : 40, multiply every linear measurement of the train by the scale factor of $\frac{1}{40}$ or 0.025. Scale factors less than 1 (such as 1 : 40, $\frac{1}{40}$, 0.025, and 2.5%) indicate a reduction. Scale factors greater than 1 (such as 3 : 2, $\frac{3}{2}$, 1.5, and 150%) indicate an enlargement.

Scale diagrams and scale models allow you to visualize or handle shapes or objects that might otherwise be too large or too small to see and manipulate.

Some objects, such as buildings, may be represented using multiple 2-D scale diagrams. For example, plans for a building allow you to compare the sizes of rooms, decide where doors should be placed, and so on, before spending money on construction.

Study *Aid*
- See Lesson 8.3, Examples 1 to 3.
- See Lesson 8.5, Examples 1 to 3.
- Try Chapter Review Questions 5 to 7 and 11 to 13.

Q: How are the areas of two similar shapes related?

A: When two shapes are similar, their corresponding dimensions are proportional. The ratio of a pair of corresponding dimensions is a number called the scale factor. The scale factor is often represented by k. The areas of two similar shapes are related by the square of the scale factor.

Area of similar shape $= k^2$(Area of original shape)

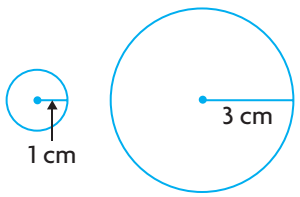
For example, these two circles are similar.

The scale factor that relates the dimensions of the small circle to those of the large circle is 3. So,

Area of large circle $= 3^2$(Area of small circle)

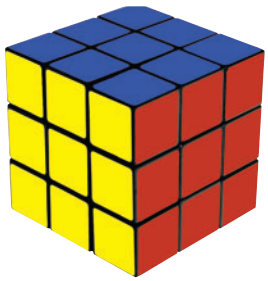
Area of large circle $= 9$(Area of small circle)

Study *Aid*
- See Lesson 8.4, Examples 1 and 2.
- Try Chapter Review Questions 8 to 10.



Study Aid

- See Lesson 8.6, Examples 1 and 2.
- Try Chapter Review Questions 14 to 16.



Q: How are the surface area and volume of similar objects related?

A: Consider the Rubik's Cube shown. It consists of 27 individual cubes, each similar to the Rubik's Cube itself. The scale factor between the length of a side of an individual cube and the length of a side of the Rubik's Cube is $\frac{3}{1}$ or 3.

The Rubik's Cube has a surface area of (6)(9) or 54 square units. Each individual cube has a surface area of 6 square units. The ratio of the surface areas is $\frac{54}{6}$ or 9. This is the value of the scale factor squared.	The Rubik's Cube has a volume of (3)(3)(3) or 27 cubic units. Each individual cube has a volume of 1 cubic unit. The ratio of the volumes is $\frac{27}{1}$ or 27. This is the value of the scale factor cubed.
The surface area of the similar object, $SA_{similar}$, is related to the original object, $SA_{original}$, by the square of the scale factor. $SA_{similar} = k^2(SA_{original})$	The volume of the similar object, $V_{similar}$, is related to the original object, $V_{original}$, by the cube of the scale factor. $V_{similar} = k^3(V_{original})$

Study Aid

- See Lesson 8.6, Example 3.
- Try Chapter Review Question 16.

Q: If you know the surface areas or volumes of two similar objects, how can you determine the scale factor that relates their dimensions?

A:

Area increases/decreases by the square of the scale factor, k, that relates the original object to the similar object. Therefore, the scale factor is the square root of the ratio of the surface areas.	Volume increases/decreases by the cube of the scale factor, k, that relates the original object to the similar object. Therefore, the scale factor is the cube root of the ratio of the volumes.
For example, in the Rubik's Cube, $k^2 = \frac{\text{Surface area of Rubik's Cube}}{\text{Surface area of small cube}}$ $k^2 = \frac{54}{6}$ $k = \sqrt{9}$ $k = 3$	For example, in the Rubik's Cube, $k^3 = \frac{\text{Volume of Rubik's Cube}}{\text{Volume of small cube}}$ $k^3 = \frac{27}{1}$ $k = \sqrt[3]{27}$ $k = 3$

PRACTISING

Lesson 8.1

1. An athlete runs the first lap of a race slightly faster than the second lap, and then runs the final lap the fastest. Draw a distance versus time graph that compares the athlete's average speed on each lap.

2. For each of the following, compare the two rates and determine the lower rate.

a) frozen hams: \$2.58/kg or \$0.226/100 g

b) cycling speeds: 35 km/h or 15 min to travel 4.5 mi

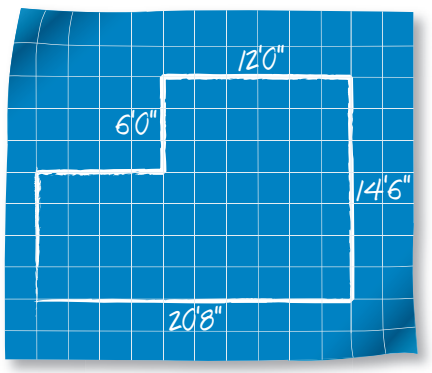
c) fuel efficiency: 6.5 L/100 km or 38 L of fuel needed to travel 560 km

d) speed of a falling object: 10 m/s or 60 km/h

Lesson 8.2

3. Based on her best consistent pace in practice, Petra believes that she can run her next marathon at an average pace of 3.75 min/km. An official marathon course is 42.195 km long. To qualify for the Boston Marathon, she must run a marathon in 3 h 40 min or better. If she manages to maintain her target pace throughout her next marathon, will Petra qualify to run in the Boston Marathon?

4. Doris wants to buy new carpet for her living room, which has the dimensions shown. She plans to order about 10% extra, so that she has enough to allow for loss during cutting. She can buy the carpet locally for \$36.95 per square yard, or she can buy it from a store in a nearby city for \$33.99 per square yard. However, the store in the nearby city charges \$100 for delivery.

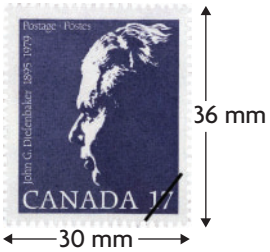


a) How much carpet should Doris buy?

b) Where should she buy the carpet? Explain.

Lesson 8.3

5. In Humboldt, Saskatchewan, there is a 2.4 m by 2.0 m reproduction of a stamp that honours John Diefenbaker, Canada's 13th prime minister. What scale factor was used to make the reproduction?



6. Find a 2-D shape in your classroom, and measure its dimensions.

a) Determine a reasonable scale factor you can use to create a scale diagram on half of a sheet of standard paper.

b) Draw a scale diagram of your shape.

7. An airplane starts at point A. It flies N30°E, at a speed of 125 mph, for 5 h to point B. Then it flies S20°E, at a speed of 100 mph, for 3 h to point C.

a) Make a scale drawing of the airplane's flight path.

b) Explain how you could estimate the distance from point C to point A without using trigonometry.

Lesson 8.4

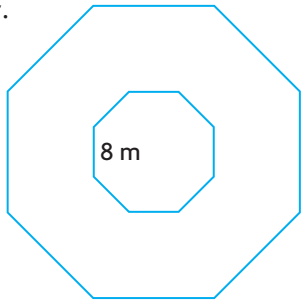
8. The owners of a local pizzeria advertise their Gynormous pizza as being 40% bigger than their competitors' pizzas. They do not say, however, what they mean by "bigger."

a) If they mean that the diameter is 40% greater, what is the percent increase in area?

b) If they mean that the area is 40% greater, what is the percent increase in diameter?

c) Which of these two meanings do you think was implied by the owners of the pizzeria? Explain.

9. The area of the larger regular octagon is exactly 2500 m^2.



a) Determine the area of the smaller octagon.

b) Determine the scale factor, to the nearest hundredth, that was used to enlarge the smaller regular octagon.

10. a) Suppose that you put a 5 in. by 7 in. picture in a copy machine and click "enlarge 110%." What will the dimensions of the copy be?

b) By what percent will the area of the picture increase?

Lesson 8.5

11. In a local store, Serena saw a toy pig with a scale ratio of 1 : 16. She estimated that the toy pig was about 10 cm long. She searched online and found a similar toy pig, with a scale ratio of 1 : 64. Estimate the length of the online toy.

12. At the Visitor Centre in 100 Mile House, British Columbia, there is a display of a giant pair of cross-country skis. An average person would use skis that are 200 cm long and poles that are 150 cm long. The giant skis are 12.0 m long.

a) Determine the scale factor that was used to create the display.

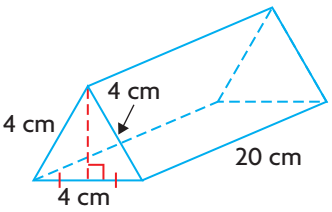
b) Determine the length of the poles in the display.

13. Jonas collects and builds model airplanes. He wants to build a 1 : 20 scale model of a floatplane. He searches the Internet for information about the real plane and learns that it has a wingspan of 36 ft, a length of 26 ft 2 in., and a height of 7 ft 6 in. Jonas wants to build a glass display case, in the form of a rectangular prism, for his scale model. He wants the dimensions of the display case to be 20% larger than each dimension of the model. Determine the dimensions of Jonas's display case.

Lesson 8.6

14. Cone A is a reduction of cone B, with a scale factor of 1 : 9. Cone A has a volume of 20 cm^3. What is the volume of cone B?

15. A chocolate bar is sold in a package, as shown. The manufacturer doubles the volume of the chocolate to create a larger bar, similar in shape to the original bar. Determine the surface area, to the nearest square centimetre, of the package that is needed for the new bar.



16. A cellphone company advertises that it has created a similar version of its most popular phone, reducing the volume and mass of the original phone by 48.8%. The original phone is a rectangular prism, 50 mm wide by 95 mm long by 10 mm high. Determine the dimensions of the new phone.